Name:_____

Teacher/Class:_____

SYDNEY TECHNICAL HIGH SCHOOL

HSC ASSESSMENT TASK 1

DECEMBER 2005

MATHEMATICS

Time Allowed: **70 minutes**

Instructions:

- Start each question on a new page.
- You may write on the front and back of each sheet of paper. Ask for more paper if required.
- Indicated marks are a guide only and may be changed slightly if necessary.
- Marks may not be awarded for careless or badly arranged work.

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Question 7	Question 8	Total
/7	/7	/7	/7	/8	/8	/8	/9	/61

QUESTION 1:

Marks

Differentiate:

i) $2x^3 + x + 4$ **1**

ii) $(5x^2 - 3)^4$ **2**

iii) $\dfrac{3x}{2x+5}$ **2**

iv) $(x-3)(x^3 + 5x - 6)$ **2**

QUESTION 2: (Start new page)

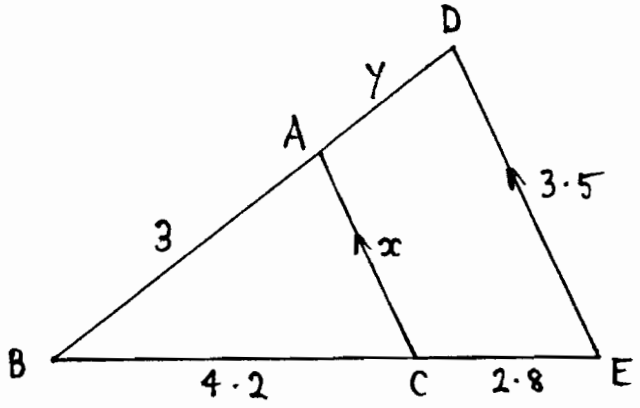
A. Show that the point (2, -8) lies on the curve $y = 3x^3 - 8x^2$ **1**

i) Hence find the equation of the tangent at this point **3**

B. Find the equation of the locus of the point $P(x, y)$
if PA = PB where A is the point (-2, 1) and B is (4, -3) **3**

QUESTION 3: (Start new page) **Marks**

A.



i) By which test is $\Delta ABC \;|||\; \Delta DBE$? **1**

(DO NOT PROVE)

ii) Hence or otherwise find the value of x and y by giving a reason and showing all working **3**

B. **i)** Another way of writing $\sqrt{2x-4}$ is $(2x - 4)^a$. Find a **1**

ii) Hence differentiate $x^2\sqrt{2x-4}$ **2**

QUESTION 4: (Start a new page)

A. Let the roots of $x^2 + 3x - 5 = 0$ be α and β. Without solving the equation, find the values of: **4**

i) $\alpha + \beta$ **ii)** $\alpha\beta$

iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ **iv)** $\alpha^2 + \beta^2$

B. By making a suitable substitution, solve $3^{2x} - 10.3^x + 9 = 0$ **3**

QUESTION 5: (Start a new page) Marks

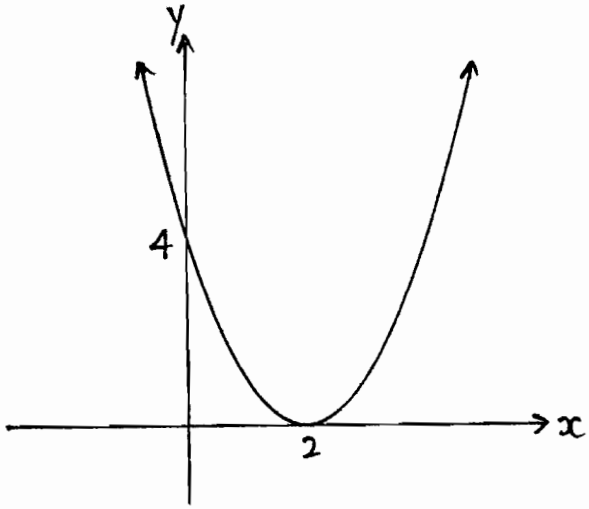
A. A parabola has an equation $y = x^2 - 6x - 7$

i) Sketch this parabola showing its vertex and the x and y intercepts **4**

ii) Hence or otherwise solve $x^2 - 6x - 7 > 0$ **1**

iii) What is the minimum value of this parabola? **1**

B. This diagram could be the graph of: **1**



(A) $y = x^2 + 4$

(B) $y = x^2 - 2x$

(C) $y = x^2 + 4x$

(D) $y = (x-2)^2$

(E) $y = (x+2)^2$

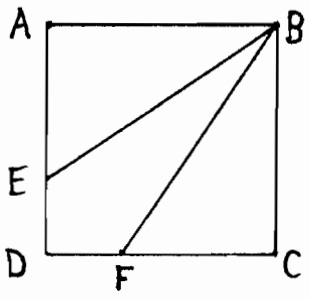
C. Sketch a negative definite quadratic function. **1**

QUESTION 6: (Start a new page)

Marks

A. Find the equation of the locus of the point $P(x, y)$ which moves so that it is always a distance of 3 units from the point (2,-1). **2**

B.



ABCD is a square

ED = FD

i) Prove that triangles BAE and BCF are congruent. **3**

ii) Given $\angle BFC = 55°$, find $\angle EBF$ giving reasons **3**

QUESTION 7: (Start a new page)

A. The vertex of a parabola is (1,4) and its directrix is $x = -3$

i) Sketch the parabola **1**

ii) Find the focal length "a" of this parabola and hence give the coordinates of the focus. **2**

iii) Find the equation of this parabola **2**

B. Find the values of a, b and c if **3**

$2x^2 + 3x - 5 \equiv a(x+1)^2 + b(x+1) + c$

QUESTION 8: (Start a new page) **Marks**

A. Find the range of values of k for which the roots of **4**

$x^2-(k+2)x+(k+5)=0$ are real

B. **i)** A normal to the curve $y=3x^2-5x+2$ has a gradient of -1 **3**

Find the coordinates of the point of intersection between the normal and the curve.

ii) Hence find where this normal cuts the y axis. **2**

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Question 6

A. A circle:

$(x-h)^2+(y-k)^2=r^2$ ①

$(x-2)^2+(y+1)^2=9$ ①

OR

$\sqrt{(x-2)^2+(y--1)^2}=3$ ①

$(x-2)^2+(y+1)^2=9$ ①

B.ci) $BA=BC$ (Sides of a square)

$\angle A=\angle C$ (Angles of a square) (

$AE=FC$ ($ED=FD$ are subtracted from equal sides) ①

$\therefore \Delta BAE \equiv \Delta BCF$ (SAS) ①

(Take off one for each error or missing reason)

Bcii) $BFC=55$

$\Rightarrow FBC=35$ (angle sum of a Δ) ①

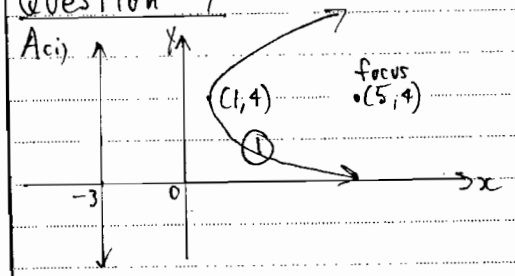
$\angle ABE=\angle FBC$ (corresponding angles in congruent triangles) ①

$\therefore \angle EBF=90-2\times 35$ (angle sum of a right angle)

$=20°$ ①

Question 7

Aci)



① ①

(ii) $a=4$ units ①

Focus is $(5,4)$ ①

(iii) Standard form:

$(y-k)^2=4a(x-h)$ ①

$(y-4)^2=16(x-1)$ ①

B. $2x^2+3x-5 \equiv a(x+1)^2+b(x+1)+c$

$=ax^2+2ax+a+bx+b+c$

$=ax^2+(2a+b)x+(a+b+c)$

$\therefore a=2$ ①, $2a+b=3$, $a+b+c=-5$

$2\times 2+b=3$, $2+-1+c=-5$

$b=-1$ ①, $c=-6$ ①

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Question 8

A $x^2-(k+2)x+(k+5)=0$

Real roots if $\Delta \geq 0$ ①

ie: $(k+2)^2-4\times 1\times(k+5)\geq 0$ ①

$k^2+4k+4-4k-20\geq 0$

$k^2-16\geq 0$ ①

$(k-4)(k+4)\geq 0$

$k\geq 4$ or $k\leq -4$ ①

B $y=3x^2-5x+2$

$\frac{dy}{dx}=6x-5$ ①

Normal gradient -1

$\Rightarrow$ Tangent gradient 1

$\Rightarrow 1=6x-5$ ①

At $x=1$ and $y=0$

ie: $(1,0)$ is pt. of intersection ①

$m=-1$, $(1,0)$

$y-0=-1(x-1)$ ①

$y=-x+1$

cuts y axis when $x=0$

ie: at $y=1$ ①

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Solutions to 2005 Mathematics Assessment Task 1

Question 1

(i) $6x^2+1$ (1)

(ii) $4(5x^2-3)^3 \times 10x$ (1)
$= 40x(5x^2-3)^3$ (1)

(iii) $\frac{d}{dx}\left(\frac{3x}{2x+5}\right)$
$= \frac{(2x+5)3 - 3x(2)}{(2x+5)^2}$ (1)
$= \frac{15}{(2x+5)^2}$ (1)

(iv) $(x-3)(3x^2+5) + x^3+5x-6$ (1)
$3x^3+5x-9x^2-15+x^3+5x-6$
$4x^3-9x^2+10x-21$ (1)

Question 2

A (i) $(2,-8)$ must satisfy $y = 3x^3 - 8x^2$
ie: $-8 = 3\times2^3 - 8\times2^2$ (1)
$-8 = 24 - 32$ (1)
$-8 = -8$ ✓

(ii) $\frac{dy}{dx} = 9x^2 - 16x$
At $x=2$, m of tangent is:
$9\times2^2 - 16\times2 = 4$ (1)
$\therefore y - -8 = 4(x-2)$ (1)
$4x - y - 16 = 0$ or $y = 4x$ (1)

B. $PA = PB$
$\sqrt{(x--2)^2+(y-1)^2} = \sqrt{(x-4)^2+(y--3)^2}$ (1)
$x^2+4x+4+y^2-2y+1 = x^2-8x+16+y^2+6y+9$ (1)
$12x - 8y - 20 = 0$
$3x - 2y - 5 = 0$ (1)

Question 3

(i) Equiangular (1)

(ii) $\frac{x}{3\cdot5} = \frac{4\cdot2}{4\cdot2+2\cdot8}$ and $\frac{y+3}{3} = \frac{4\cdot2+2\cdot8}{4\cdot2}$
$x = 2\cdot1$ (1) and $4\cdot2(y+3) = 21$
$y = 2$ (1)

Ratio of intercepts theorem OR
Corresponding sides of similar triangles make the same rati (1)

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B (i) $\sqrt{2x-4} = (2x-4)^{\frac{1}{2}}$
$\therefore a = \frac{1}{2}$ (1)

(ii) $\frac{d}{dx}\left[x^2(2x-4)^{\frac{1}{2}}\right]$
$= x^2 \times \frac{1}{2}(2x-4)^{-\frac{1}{2}} \times 2 + 2x$
$= \frac{x^2}{\sqrt{2x-4}} + 2x\sqrt{2x-4}$

Question 4

A. (i) $\alpha+\beta = \frac{-b}{a}$
$= -3$ (1)

(ii) $\alpha\beta = \frac{c}{a}$
$= -5$ (1)

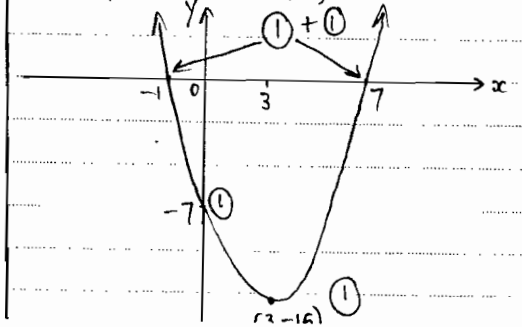
(iii) $\frac{1}{\alpha}+\frac{1}{\beta}$
$= \frac{\alpha+\beta}{\alpha\beta}$
$= \frac{-3}{-5}$
$= \frac{3}{5}$ (1)

(iv) α
$= ($
$=$
$=$

B. $3^{2x} - 10\times3^x + 9 = 0$
Let $u = 3^x$
$\therefore u^2 - 10u + 9 = 0$ (1)
$(u-1)(u-9) = 0$
$u = 1$ or $u = 9$ (1)
$3^x = 1$ or $3^x = 9$
$x = 0$ or 2 (1)

Question 5

A. (i) $y = x^2 - 6x - 7$
$(0,-7)$ y intercept
$0 = x^2 - 6x - 7$
$0 = (x+1)(x-7)$
$\Rightarrow (-1,0)$ and $(7,0)$



(ii) $x^2 - 6x - 7 > 0$
$x < -1, \quad x > 7$

(iii) Minimum value

B Answer D

C.